

# Trigonometric Identities

## Reciprocal:

$$\begin{aligned}\sin \theta &= \frac{1}{\csc \theta} \\ \cos \theta &= \frac{1}{\sec \theta} \\ \tan \theta &= \frac{1}{\cot \theta}\end{aligned}$$

## Ratio:

$$\begin{aligned}\tan \theta &= \frac{\sin \theta}{\cos \theta} \\ \tan \theta &= \frac{\sec \theta}{\csc \theta} \\ \cot \theta &= \frac{\cos \theta}{\sin \theta}\end{aligned}$$

## Sum and Difference:

$$\begin{aligned}\cos(\alpha \pm \beta) &= \cos \alpha \cos \beta \mp \sin \alpha \sin \beta \\ \sin(\alpha \pm \beta) &= \sin \alpha \cos \beta \pm \cos \alpha \sin \beta \\ \tan(\alpha \pm \beta) &= \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}\end{aligned}$$

## Pythagorean:

$$\begin{aligned}\sin^2 \theta + \cos^2 \theta &= 1 \\ \tan^2 \theta + 1 &= \sec^2 \theta \\ 1 + \cot^2 \theta &= \csc^2 \theta\end{aligned}$$

## Symmetry:

$$\begin{aligned}\sin(-\theta) &= -\sin \theta \\ \cos(-\theta) &= \cos \theta \\ \tan(-\theta) &= -\tan \theta\end{aligned}$$

## Product-to-Sum:

$$\begin{aligned}\cos \alpha \cos \beta &= \frac{1}{2} (\cos(\alpha - \beta) + \cos(\alpha + \beta)) \\ \cos \alpha \sin \beta &= \frac{1}{2} (\sin(\alpha + \beta) - \sin(\alpha - \beta)) \\ \sin \alpha \sin \beta &= \frac{1}{2} (\cos(\alpha - \beta) - \cos(\alpha + \beta)) \\ \sin \alpha \cos \beta &= \frac{1}{2} (\sin(\alpha + \beta) + \sin(\alpha - \beta))\end{aligned}$$

## Sum-to-Product:

$$\begin{aligned}\cos u + \cos v &= 2 \cos \left( \frac{u+v}{2} \right) \cos \left( \frac{u-v}{2} \right) \\ \sin u - \sin v &= 2 \cos \left( \frac{u+v}{2} \right) \sin \left( \frac{u-v}{2} \right) \\ \sin u + \sin v &= 2 \sin \left( \frac{u+v}{2} \right) \cos \left( \frac{u-v}{2} \right) \\ \cos u - \cos v &= -2 \sin \left( \frac{u+v}{2} \right) \sin \left( \frac{u-v}{2} \right)\end{aligned}$$

## Cofunction:

$$\begin{aligned}\cos \left( \frac{\pi}{2} - t \right) &= \sin t \\ \sin \left( \frac{\pi}{2} - t \right) &= \cos t\end{aligned}$$

## Double-Angle:

$$\begin{aligned}\cos(2\alpha) &= \cos^2 \alpha - \sin^2 \alpha \\ &= 1 - 2 \sin^2 \alpha \\ &= 2 \cos^2 \alpha - 1 \\ \sin(2\alpha) &= 2 \sin \alpha \cos \alpha \\ \tan(2\alpha) &= \frac{2 \tan \alpha}{1 - \tan^2 \alpha}\end{aligned}$$

## Power-Reduction:

$$\begin{aligned}\cos^2 \alpha &= \frac{1 + \cos(2\alpha)}{2} \\ \sin^2 \alpha &= \frac{1 - \cos(2\alpha)}{2} \\ \tan^2 \alpha &= \frac{1 - \cos(2\alpha)}{1 + \cos(2\alpha)}\end{aligned}$$

## Half-Angle:

$$\begin{aligned}\cos \left( \frac{u}{2} \right) &= \pm \sqrt{\frac{1 + \cos u}{2}} \\ \sin \left( \frac{u}{2} \right) &= \pm \sqrt{\frac{1 - \cos u}{2}} \\ \tan \left( \frac{u}{2} \right) &= \pm \sqrt{\frac{1 - \cos u}{1 + \cos u}} \\ &= \frac{1 - \cos u}{\sin u} \\ &= \frac{\sin u}{1 + \cos u}\end{aligned}$$

Also visit [http://en.wikipedia.org/wiki/List\\_of\\_trigonometric\\_identities](http://en.wikipedia.org/wiki/List_of_trigonometric_identities)  
and [http://en.wikipedia.org/wiki/Trigonometric\\_functions](http://en.wikipedia.org/wiki/Trigonometric_functions) for more information.